A

20231 **120 MINUTES**

1. Which of the following statements is /are true:
    1. Every Cauchy sequence is bounded
    2. Every bounded sequence is always a Cauchy sequence
    3. A sequence converges in real line if and only if it is a Cauchy sequence

    A) 2 & 3 only B) 1 & 3 only C) All D) none

2. $\sum_{n=1}^{\infty} \frac{1}{n^{\alpha+\frac{1}{n}}}$ is:
    A) always divergent
    B) always convergent
    C) convergent if $\alpha > 1$ and divergent if $\alpha \leq 1$
    D) convergent if $\alpha \leq 1$ and divergent if $\alpha > 1$

3. If [x] denote the greatest integer not greater than x, then $\lim_{x\to 0}[x] =$
    A) 0 B) -1 C) 1 D) does not exist

4. The function on R defined by $f(x) = 2x + |x|$ is:
    A) differentiable and continuous
    B) not differentiable but continuous
    C) differentiable but not continuous
    D) neither differentiable nor continuous

5. Which of the following is not true in a metric space?
    A) finite union of open sets is open
    B) finite intersection of open sets is open
    C) arbitrary union of open sets is open
    D) arbitrary intersection of open sets is open

6. A bounded function f is Riemann integrable on [a, b] if the set of its points of discontinuity is:
    A) finite B) infinite C) oscillatory D) none of these

7. If Lebesgue outer measure of a set E is 0, then
    A) E is measurable B) E is not measurable
    C) E is always empty D) none of the above

8. The set {(2, -1, 3), (3, 4, -1), (k, 2, 1)} is linearly dependent if:
    A) $k = 3$ B) $k = -1$ C) $k = 0$ D) $k \neq 3$

9. If A is a square matrix, then $A + A^T$ is a:
    A) symmetric matrix B) skew symmetric matrix
    C) idempotent matrix D) none of these

10. Let A be an m × n matrix. Then

A) Rank (A) ≤ Rank (AA^T) B) Rank (A) ≥ Rank (AA^T)

C) Rank (A) = Rank (AA^T) D) Rank (A) ≤ Rank ($A^T A$)

11. Each eigen value of an idempotent matrix is:

A) 1 B) either 0 or 1

C) 0 D) none of these

12. The quadratic form $x^2 + 6xy + 10y^2$ is:

A) positive definite B) positive semi-definite

C) negative definite D) indefinite

13. If X denotes the number of tosses required to obtain a head in tossing an unbiased coin, then the pgf P(s) of X is:

A) $\frac{1}{2-s}$ B) $\frac{s}{2-s}$ C) $\frac{1}{1-s}$ D) none of these

14. Let $X_1, X_2, \ldots, X_n$ be iid random variables with mean 0 and variance 1. Then $P[|X_1 + X_2 + \cdots X_n| \geq \sqrt{n}]$

A) ≤ 1 B) $\leq n^{-1}$ C) = 0 D) none of these

15. Let X and Y be two random variables with common mean and common variance. Then X + Y and X - Y are

A) independent B) uncorrelated and not necessarily be independent

C) correlated D) none of these

16. Let A and B are two independent events defined on some probability space with $P(A) = \frac{1}{3}$ and $P(B) = \frac{3}{4}$. Then $P\{A|AUB\} =$

A) $\frac{1}{3}$ B) $\frac{5}{6}$ C) $\frac{2}{5}$ D) 1

17. For any two events A and B

A) $P(A \cap B) \geq 1 - P(A^c) - P(B^c)$ B) $P(A \cap B) \geq 1 - P(A) - P(B^c)$

C) $P(A \cap B) \geq 1 - P(A) - P(B)$ D) $P(A \cap B) \leq 1 - P(A^c) - P(B^c)$

18. Let $\{A_n\}$ be a non increasing sequence of events in the sample space. Then $\lim_{n\to\infty} P(A_n) =$

A) $P(\cap_{n=1}^{\infty} A_n)$ B) $P(\cup_{n=1}^{\infty} A_n)$ C) 0 D) 1

19. If ordered samples of size 'r' are drawn from a population of 'n' elements without replacement, the total number of possible samples is:

A) n^r B) nC_r C) nP_r D) r!

20. The set of discontinuity points of a distribution function is

A) uncountable B) at most countable

C) finite D) none of these

21. Let G be a function of two variables defined by G(x, y) =1 if $x + 2y \geq 1$, and 0 if $x + 2y < 1$. Then G is:

A) distribution function of a pair of mixed random variables
B) distribution function of a pair of continuous random variables
C) distribution function of a pair of discrete random variables
D) not a distribution function

22. Which of the following is true for the random variables X and Y having joint pdf $f(x,y) = \frac{1+xy}{4}, |x| < 1, |y| < 1$.

1. X and Y are independent
2. X^2 and Y^2 are independent

A) Only 1 is true B) Only 2 is true
C) Both 1 and 2 are true D) None of these

23. The numbers 1, 2, …, 20 are arranged in random order. The probability that the digits 1, 2, …, 12 appear as neighbours in that order is:

A) $\frac{3}{5}$ B) $\frac{8!}{20!}$ C) $\frac{12!\times 8!}{20!}$ D) $\frac{9!}{20!}$

24. There are two bags. One bag contains 4 red and 5 black balls and the other 5 red and 4 black balls. One ball is drawn from either of the two bags. Probability of the drawing a black ball is:

A) 1 B) $\frac{16}{81}$ C) $\frac{1}{2}$ D) none

25. Let X and Y be two random variables with $E(X^2) = 0.5$ and $E(Y^2) = 0.2$. Then

A) $E[(XY)^2] = 0.1$ B) $E[(XY)^2] \geq 0.1$
C) $E[(XY)^2] \leq 0.1$ D) $E[(XY)^2] = 0.01$

26. Let $X \sim P(\lambda)$, where λ is a positive integer. Then

A) the distribution is unimodal
B) the distribution is bimodal
C) the distribution has no mode
D) the distribution has negative mode

27. Let $X_1, X_2, ..., X_n$ be iid random variables that follow geometric distribution with parameter p. Then which of the following statement is true?

A) $\min(X_1, X_2, ..., X_n)$ is a geometric random variable with parameter $1-(1-p)^n$
B) $\min(X_1, X_2, ..., X_n)$ is a geometric random variable with parameter $(1-p)^n$
C) $\min(X_1, X_2, ..., X_n)$ is a negative binomial random variable with probability of success $1-(1-p)^n$
D) $\min(X_1, X_2, ..., X_n)$ is a negative binomial random variable with probability of success $(1-p)^n$

28. A box contains 20 marbles. Of these, 12 are drawn at random, marked and returned to the box. The content of the box is thoroughly mixed and 5 marbles are drawn at random from the box without replacement. Then the mean number of marked marbles in the sample is:

A) 2 B) 3 C) 4 D) 5

29. If X follows exponential distribution with mean β, then the distribution of $Y = 1 - e^{-\beta X}$ is:

A) $U[0, 1]$ B) exponential C) Weibull D) Pareto

30. The moment generating function $M(t)$ of gamma distribution with pdf

$$f(y) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} exp\left(-\frac{y}{\beta}\right) y^{\alpha-1}, y > 0, \alpha, \beta > 0.$$

exists when

A) $t < \beta$ B) $t > \beta$ C) $t < \frac{1}{\beta}$ D) $t > \frac{1}{\beta}$

31. Let X and Y be iid $N(0, \sigma^2)$ random variables. Then the distribution of $\frac{X}{Y}$ is

A) Normal B) Cauchy
C) Chi-square distribution D) F-distribution

32. Which of the following distribution has positive support

A) logistic distribution
B) Weibull distribution
C) double exponential distribution
D) normal distribution

33. In terms of the incomplete beta function defined as $I_u(x, y) = \frac{\int_0^u t^{x-1}(1-t)^{y-1} dt}{\int_0^1 t^{x-1}(1-t)^{y-1} dt}$, the distribution function of the r^{th} order statistic $X_{r:n}$ when the sample is taken from the distribution function $F(x)$ is:

A) $I_{F(x)}(r-1, n-r)$ B) $I_{F(x)}(r-1, n-r+1)$
C) $I_{F(x)}(r, n-r+1)$ D) $I_{F(x)}(r, n-r)$

34. Let $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ be the order statistics of a random sample taken from $U[0, 1]$ population. The $E(X_{r:n}) =$

A) $\frac{r}{n}$ B) $\frac{r-1}{n}$ C) $\frac{r-1}{n+1}$ D) $\frac{r}{n+1}$

35. Let X ~ F(m, n). Then $E(X)$ exists only when

A) $m > 1$ B) $n > 1$ C) $m > 2$ D) $n > 2$

36. Let $X_1, X_2, ..., X_n$ be independent $N(\mu, 1)$ random variables, then the distribution of $Y = \sum_{i=1}^{n} X_i^2$ follows:

A) Central Chi-square distribution
B) t - distribution
C) F -distribution
D) None of these

37. Which among the following statement(s) on bivariate distributions is/are true

1. If (X, Y) has bivariate normal distribution, X and Y are independent if and only if the correlation between X and Y is zero
2. If the marginal distribution of X and Y are normal then the joint distribution of (X, Y) is always bivariate normal

A) 1 only B) 2 only C) 1 and 2 D) None of these

38. If $E(X^2) < \infty$, then

A) $V(X) \leq V[E(X|Y)]$ B) $V(X) \geq V[E(X|Y)]$

C) $V(X) = V[E(X|Y)]$ D) $V(X) = V[E(Y|X)]$

39. Let $X_1, X_2, \ldots, X_n$ be a random sample from $b(1, \theta)$ and X be the number of 1's in the sample. Then an unbiased estimator of θ^2 is

A) $\left(\frac{X}{n}\right)^2$ B) $\frac{nX - X^2}{n(n-1)}$ C) $\frac{X(X-1)}{n(n-1)}$ D) None of these

40. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, \theta^2)$. Then which of the following is true?

A) $\sum X_i$ is a complete sufficient estimator of θ
B) $\sum X_i^2$ is a complete sufficient estimator of θ
C) $\left(\sum X_i, \sum X_i^2\right)$ is a complete sufficient estimator of θ
D) $\left(\sum X_i, \sum X_i^2\right)$ is a sufficient estimator of θ but not complete

41. Which among the following statement(s) is/are TRUE?

1. A complete sufficient statistic is minimal sufficient.
2. A minimal sufficient statistic is complete sufficient
3. A statistic that is independent of every ancillary statistic is always complete

A) 1 only B) 2 only C) 1 and 3 only D) 2 and 3 only

42. Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean β . Then the UMVUE of β is

A) $\bar{X}$ B) $\frac{1}{\bar{X}}$ C) $X_{(1)}$ D) $nX_{(1)}$

43. Which among the following PDFs satisfy the regularity conditions of Cramer-Rao lower bound?

1. $f_\theta(x) = \theta^{-1} e^{-x/\theta}, x > 0, \theta > 0$ 2. $f_\theta(x) = e^{-(x-\theta)}, x > \theta$

3. $f_\theta(x) = \frac{1}{\theta}, 0 < x < \theta$ 4. $f_\theta(x) = \theta(1-\theta)^x, x = 0, 1, ...; 0 < \theta < 1$

A) 1 & 2 only B) 1 & 4 only C) 1, 2 & 4 only D) 1, 2, 3 & 4

44. Based on a random sample of size n from $U(-\theta, \theta)$, the MLE of θ is

A) $\max(X_i)$ B) $\min(X_i)$

C) $\max(X_i) + \min(X_i)$ D) $\max(|X_i|)$

45. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from $P(\lambda)$. Suppose that the prior distribution of λ is taken as gamma distribution with parameter (α, β). Then posterior distribution of λ is

A) Beta distribution of first type B) Beta distribution of second type

C) Gamma distribution D) None of these

46. A random sample of size n is taken from the pdf $f_\theta(x) = \frac{\theta}{x^{\theta+1}}, x \geq 1, \theta > 1$. Then the estimate of θ by the method of moments is

A) $\bar{X}$ B) $\frac{\bar{X}-1}{\bar{X}}$ C) $\frac{\bar{X}}{1-\bar{X}}$ D) $\frac{\bar{X}}{\bar{X}-1}$

47. Let α and β be the probabilities of type I and type II errors of the most powerful test for testing a simple null hypothesis against a simple alternative hypothesis. If $0 < \alpha < 1$, then

A) $\alpha < \beta$ B) $\alpha < 1 - \beta$ C) $\alpha \leq \beta$ D) $\alpha \leq 1 - \beta$

48. Let $X_1, X_2, \ldots, X_n$ be a random sample from the Cauchy distribution with pdf

$f_\theta(x) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}, x \in R$. Then which of the following statement is true.

A) $\{f_\theta(x)\}$ possess MLR property

B) Uniformly most powerful test exists for testing H$_0$: $\theta \leq 0$ against $H_1: \theta > 0$.

C) The critical region of a locally most powerful test for testing H$_0$: $\theta \leq 0$ against $H_1: \theta > 0$ has the form $\sum_{i=1}^{n} \frac{2x_i}{1+x_i^2} > k$.

D) All the above statements are true

49. Let ℓ be the likelihood ratio statistic for testing the hypothesis $H_0: \theta \in \Theta_0$ against $H_1: \theta \in \Theta_1$. Then under some regularity conditions, the asymptotic distribution of $-2\log \ell$ is:

A) Normal B) Student's- t C) Chi-square D) None of these

50. p - value of a test is the:

A) largest significance level at which the null hypothesis cannot be rejected
B) smallest significance level at which the null hypothesis cannot be rejected
C) largest significance level at which the null hypothesis can be rejected
D) smallest significance level at which the null hypothesis can be rejected

51. Operating Characteristic (OC) function is:

A) the probability of the sequential test procedure terminating with the acceptance of the null hypothesis
B) the probability of the sequential test procedure terminating with the rejection of the null hypothesis
C) the probability of the sequential test procedure arriving at a conclusion
D) None of these

52. The Mann-Whitney U test is preferred to a t-test when

A) data are paired
B) sample sizes are small
C) the assumption of normality is not met
D) sample is dependent

53. A certain steel bar is measured with a device which has a known precision σ = 0.5mm. Suppose we want to estimate the mean 'measurement' with an error at most 0.2mm at the level of significance α = 0.05. What sample size is required if we assume normality?

A) 6 B) 7 C) 24 D) 25

54. In simple random sampling of n units from N units without replacement, the probability of a specified unit of the population being included in the sample is:

A) $\frac{1}{NC_n}$ B) $\frac{1}{n}$ C) $\frac{1}{N}$ D) $\frac{n}{N}$

55. In simple random sampling of n units from N units, the variance of the sample mean in SRSWOR is exactly half of the variance of the sample mean in SRSWR when:

A) n = N B) 2n = N C) 2n = N + 1 D) n > 50

56. With the usual notations, in PPSWR an unbiased estimator for the population mean is:

A) $\frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{P_i}$ B) $\sum_{i=1}^{n}\frac{y_i}{P_i}$ C) $\frac{1}{nN}\sum_{i=1}^{n}\frac{y_i}{P_i}$ D) $\sum_{i=1}^{n}\frac{y_i}{n}$

57. A population divided into two strata having sizes 16 and 48 respectively. A random sample of size 24 is selected under proportional allocation. Then the stratum sample sizes are

A) 12, 12 B) 6, 18 C) 16, 8 D) 3, 21

58. For a population having N units systematic sampling will be more efficient than SRSWOR if the intra class correlation coefficient ρ is:

A) less than $\frac{1}{N-1}$ B) less than $-\frac{1}{N-1}$
C) less than $\frac{1}{N}$ D) greater than $-\frac{1}{N-1}$

59. Which of the following is true?

1. Ratio estimator is as good as regression estimator if regression of y on x is linear and passes through the origin.
2. Ratio estimator and regression estimators are unbiased

A) only 1 is true B) only 2 is true
C) both 1 and 2 are true D) neither 1 nor 2 is true

60. The results of a two-factor analysis of variance produce degrees of freedom (2, 24) for the F-ratio for the factor A. Based on this information, how many levels of factor A were compared in the study?

A) 2 B) 3 C) 24 D) 25

61. In Gauss-Markov theorem, which of the following is NOT an assumption on errors.

A) The errors need to be homoscedastic with finite variance
B) The errors are normally distributed
C) The errors are uncorrelated
D) None of these

62. Which of the following is TRUE?

A) Randomized block and latin square designs are connected
B) Randomized block design is connected but latin square design is not connected
C) Latin square design is connected but randomized block design is not connected
D) Randomized block and latin square designs are not connected

63. In k × k Graeco-Latin square design, the degrees of freedom of the error sum of square is equal to:

A) (k-1) B) k (k-2) C) (k-2) (k-1) D) (k-3) (k-2)

64. If N is the incidence matrix of a BIBD with usual parameters (v, b, r, k, λ) then which among the following statement(s) is/are true.

1. $NN' = (r - \lambda)I + \lambda J$, where I is $v \times v$ identity matrix and J is $v \times v$ matrix of 1's
2. Rank $(NN') = v$

A) 1 only B) 2 only C) 1 and 2 D) None of these

65. With usual notation, the estimate of the main effect of factor A in a 2^3 design in which each treatment combination is replicated n times is

A) $\frac{1}{4n}[a + ab + ac + abc - (1) - b - c - bc]$

B) $\frac{1}{4n}[b + ab + bc + abc - (1) - a - c - ac]$

C) $\frac{1}{4n}[c + ac + bc + abc - (1) - a - b - ab]$

D) $\frac{1}{4n}[abc + ab + c + (1) - a - b - bc - ac]$

66. A 2^4 design is arranged in 4 blocks by confounding the factors ABC and ACD. Then which of the following effect is also confounded with blocks.

A) AC B) BD C) BCD D) ABCD

67. Combining two or more overlapping series of index numbers having different base periods into a single continuous series is known as

A) deflating B) splicing
C) moving average D) none of these

68. If L(p) and P(q) represent respectively the Laspeyre's index number for prices and Paasche's index number for quantities, then

A) L(p) P(q) = L(q) P(p) B) L(p) P(p) = L(q) P(q)
C) L(p) P(q) = L(q) D) L(p) P(q) = P(p)

69. Which of the following is a Markov Process?

A) Poisson process B) Galton Watson branching process
C) Random walk D) All the above

70. Which of the following statements is/ are true?

1. Poisson process is a process with stationary independent increment
2. Brownian motion process is a process with stationary independent increment

A) Only (1) is true B) Only (2) is true
C) Both (1) and (2) are true D) neither (1) nor (2) are true

71. If $p_{ij}^{(n)}$ denote the n step transition probability of a Markov chain from state i to j, then $\sum_j p_{ij}^{(n)} =$

A) 1 B) 0 C) ∞ D) none of these

72. Consider one dimensional random walk on positive and negative integers, where at each transition the particle moves with probability p one unit to the right and with probability q one unit to the left such that p + q = 1. It is recurrent iff

A) $p = q = 0.5$ B) $p < q$ C) $p > q$ D) $p \neq q$

73. Let $\{X_1(t)\}$ and $\{X_2(t)\}$ be two independent Poisson processes with parameters λ_1 and λ_2 respectively. Then the distribution of $X_1(t)= k / X_1(t) + X_2(t) = n$ is:

A) Poisson with parameter $\frac{\lambda_1}{\lambda_1+\lambda_2}$
B) Binomial with parameters k and $\frac{\lambda_1}{\lambda_1+\lambda_2}$
C) Binomial with parameters n and $\frac{\lambda_1}{\lambda_1+\lambda_2}$
D) Binomial with parameters k and λ_1.

74. Among the following methods of finding trend, the method having greater subjectivity is:
A) moving average method B) free hand method
C) least squares method D) semi-average method

75. The value one season expressed as the percentage of the preceding season is known as:
A) link relatives B) Chain relatives
C) seasonal indices D) None of the above

76. In an M/M/1 queue with arrival rate λ and service rate μ, the steady state probabilities:
A) always exist B) exist only when $\lambda \leq \mu$
C) exist only when $\lambda < \mu$ D) exist only when $\lambda > \mu$

77. Which of the following are true for a random vector X having multivariate normal distribution?
1. Linear combinations of the components of X are normally distributed
2. All subsets of the components of X have a normal distribution
3. Zero covariance implies that the corresponding components are independently distributed
4. The conditional distributions of the components are normal.

A) All the above B) only 1, 2 and 4
C) only 1 and 4 D) only 1, 3 and 4

78. If X is distributed as $N_p(\mu, \Sigma)$ with $|\Sigma| > 0$, then the distribution of $(X-\mu)'\Sigma^{-1}(X-\mu)$ is distributed as:

A) χ^2 with p degrees of freedom

B) χ^2 with p - 1 degrees of freedom

C) $N_p(0, I)$

D) $N_p(0, \mu\Sigma\mu')$

79. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N_p(\mu, \Sigma)$. Define $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $S = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})'$. Then (n – 1) S is distributed as:

A) Wishart with n d.f B) Chi-square with p d.f
C) $N_p(\mu, \Sigma)$ D) Wishart with (n – 1)d.f

80. A measure of linear association of a variable X_1, with several other variables $X_2, X_3, \ldots, X_k$ is known as:
A) partial correlation B) multiple correlation
C) simple correlation D) autocorrelation