A

22121 **120 MINUTES**

1. Let A, B and C be non-empty sets and let $X = (A - B) - C$ and $Y = (A - C) - (B - C)$.Which of the following is TRUE ?

   A) $X \subset Y$ B) $X = Y$ C) $X \supset Y$ D) None of these

2. The distance of the plane $\vec{r}.(2i + 3j - 6k) + 2 = 0$ from the origin is

   A) 2 B) 14 C) $\frac{2}{7}$ D) $-\frac{2}{7}$

3. If $f(x)$ is differentiable in the interval $(2,5)$ where $f(2) = \frac{1}{5}$ and $f(5) = \frac{1}{2}$, then there exist a number c , $2 < c < 5$ for which $f'(c)$ is

   A) $\frac{1}{2}$ B) $\frac{1}{5}$ C) $\frac{1}{10}$ D) 10

4. Two independent events E and F are such that $P(E \cap F) = \frac{1}{6}$ and $P(E^c \cap F^c) = \frac{1}{3}$, $P(E) > P(F)$. Then $P(E)$ is

   A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$

5. How many four digit even numbers have all four digits distinct ?

   A) 2240 B) 2296 C) 2620 D) 4536

6. Which of the following is NOT TRUE ?

   A) If f is differentiable at a point , then f is continuous at that point .

   B) If f is differentiable at a point , then $|f|$ is also differentiable there .

   C) If $|f|$ is differentiable at a point , then it need not be true that f is differentiable there.

   D) If f is differentiable at a point , then $\frac{1}{f}$ is also differentiable at c , provided $f(c) \neq 0$.

7. Which of the following series converge ?

   A) $\sum_{n=1}^{\infty} \sin n$ B) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ C) $\sum_{n=1}^{\infty} \frac{1}{n!}$ D) $\sum_{n=1}^{\infty} \log \frac{n+1}{n}$

8. The integral $\int_0^3 [x]dx$ where $[x]$ is the greatest integer less than or equal to x is

   A) 0 B) 1 C) 2 D) 3

9. Which of the following functions is NOT of bounded variation

A) $f(x) = x^2 + x + 1$ for $x \in (-1,1)$

B) $f(x) = tan\left(\frac{\pi x}{2}\right)$ for $x \in (-1,1)$

C) $f(x) = sin\left(\frac{x}{2}\right)$ for $x \in (-\pi, \pi)$

D) $f(x) = \sqrt{1 - x^2}$ for $x \in (-1,1)$

10. Consider a function $f(z) = u + iv$ defined on $|z - i| < 1$ where u and v are real valued functions of $, y$. Then $f(z)$ is analytic for

A) $u = x^2 + y^2$ C) $u = \ln(x^2 + y^2)$

B) $u = e^{xy}$ D) $u = e^{x^2 - y^2}$

11. The residue of $f(z) = \frac{e^{2z}}{(z+1)^2}$ at $z = -1$ is

A) $2e$ B) $\frac{2}{e}$ C) $\frac{2}{e^2}$ D) e

12. The radius of convergence of the series $\sum_{n=1}^{\infty} 2^{-n} z^{2n}$ is

A) 1 B) $\sqrt{2}$ C) 2 D) ∞

13. Let $(G,*)$ be an abelian group.Then which of the following is TRUE for G ?

A) $g = g^{-1}$ for all $g \in G$. C) $(g * h)^2 = g^2 * h^2$ for all $g, h \in G$.

B) $g = g^2$ for all $g \in G$. D) G is of finite order

14. If $f: G \to G'$ is a homomorphism and e, e' are identity elements of G and G' respectively .Then which of the following is TRUE ?

A) $f(e) = e'$ C) $f(x^n) = (f(x))^n$

B) $f(x^{-1}) = (f(x))^{-1}$ D) All of these

15. Which of the following statements is NOT TRUE about Integral Domain.

A) For a given prime , the ring $(Z_p, +_p, \cdot_p)$ is an Integral Domain .

B) Every field is an Integral Domain .

C) A commutative ring R with unity is an Integral Domain if and only

if $ab = 0$, $a, b \in \mathbb{R}$, $a \neq 0$ implies $= 0$.

D) Every Integral Domain is a Field .

16. Let $I = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$ be the subring of the ring $(M_2(\mathbb{Z}), +, .)$, where $M_2(\mathbb{Z})$ denote the set of all 2×2 matrices whose entries are elements from $\mathbb{Z}$. Which of the following is TRUE ?

A) I is an ideal of $\mathbb{R}$.

B) I is a left ideal but not a right ideal of $\mathbb{R}$.

C) I is a right ideal but not a left ideal of $\mathbb{R}$.

D) I is neither a left ideal nor a right ideal of $\mathbb{R}$.

17. For an ideal I of a ring , the mapping $\gamma: R \to R/I$ be defined by $\gamma(a) = a + I , a \in R$ .Then which of the following is TRUE ?

A) $\gamma(a+b) = \gamma(a) + \gamma(b) , \forall\, a, b \in R$

B) $\gamma: R \to R/I$ is onto

C) γ is a homomorphism

D) All of these

18. The polynomial $p(x) = x^3 - x - 1$ defined over $\mathbb{Q}$ is

A) Irreducible over $\mathbb{Q}$

B) Reducible over $\mathbb{Q}$

C) Reducible over $\mathbb{Z}$

D) reducible over $\mathbb{N}$

19. $Q(\sqrt{2}, \sqrt{3})$ is the splitting field of

A) $x^2 - 2$ B) $(x^2-2)(x^2-3)$ C) $x^2 - 3$ D) $(x-2)(x-3)$

20. The number of elements in the field $\frac{Z_2[x]}{<x^3+x^2+1>}$ is

A) 2 B) 4 C) 8 D) 16

21. The rank of the matrix $\begin{pmatrix} 3 & 2 & 5 \\ -1 & 0 & 2 \\ 11 & 6 & 11 \end{pmatrix}$ is

A) 0 B) 1 C) 2 D) 3

22. The system of linear equations : $x - 2y + 3z = -2 ; -x + y - 2z = 3 ; 2x - y + 3z = 1$ has

A) Unique solution

B) No solution

C) Infinite solution

D) None of these

23. Which of the following subsets of the vectorspace $\mathbb{R}^3$ over $\mathbb{R}$ is a subspace ?

A) $W = \{(x_1, x_2, x_3) \mid x_3 = 1\}$

B) $W = \{(x_1, x_2, x_3) \mid x_3 = 0\}$

C) $W = \{(x_1, x_2, x_3) \mid x_2 > 0\}$

D) None of these

24. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(1,2) = (2,3)$ and $T(0,1) = (1,4)$. Then $T(5,6)$ is

A) (6,-1)

B) (-1,6)

C) (-6,1)

D) (1,-6)

25. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (x + y, y + z, z + x)$ for all $(x, y, z) \in \mathbb{R}^3$ .Then

A) Rank(T)=0 and Nullity(T)=3 C) Rank(T)=1 and Nullity(T)=2

B) Rank(T)=2 and Nullity(T)=1 D) Rank(T)=3 and Nullity(T)=0

26. The integrating factor of $\frac{dy}{dx} + (\tan x)y = cos^2 x$ is

A) $\cos x$ C) $\sec x$

B) $-\cos x$ D) $-\sec x$

27. The *orthogonal* trajectory of the curve $xy = c$ is

A) $x^2 - y^2 = k$ C) $x^2 + y^2 = k$

B) $2x^2 - y^2 = k$ D) None of these

28. The differential equation for variation of the amount of salt x in a tank with time t is given by $\frac{dx}{dt} + \frac{x}{20} = 10$ where x is in kg and t is in minutes . Assuming there is no salt in tank initially, the time t in which the amount of salt increases to 100kg is

A) $10 \ln 2$ C) $20 \ln 2$

B) $50 \ln 2$ D) $100 \ln 2$

29. The partial differential equation of all spheres whose centre lie on the $z - axis$ is

A) $py = qx$ C) $px = qy$

B) $px + qy = 0$ D) $py + qx = 0$

30. The number of integer less than 200 and relatively prime to it is

A) 98 B) 100 C) 101 D) 102

31. If $a \equiv b(mod\ m)$ means $a - b$ is a multiple of m ,then which of the following is NOT TRUE?

A) $12^{25} \equiv 2(mod\ 5)$ C) $13^{121} \equiv 2(mod\ 11)$

B) $8^{36} \equiv 2(mod\ 6)$ D) $9^{49} \equiv 2(mod\ 7)$

32. If $a \in \mathbb{Z}$ and p is a prime not dividing a then p divides

A) $a^{p-1} - 1$ C) $a^p - 1$

B) $a^{p+1} - 1$ D) $a^{p+2} - 1$

33. Let $f: X \rightarrow Y$ be a closed bijective map between metric spaces X and Y such that Y is compact then :

A) X need not be compact but f is continuous

B) X is compact but f need not be continuous

C) X need not be compact and f need not be continuous

D) X is compact and f is continuous

34. Which of the following is not a topological property

A) Openness C) Compactness

B) Closedness D) Boundedness

35. If X is a finite set then the cofinite topology on X is

A) Discrete C) Empty set

B) Indiscrete D) None of these

36. Let H be a Hilbert space and let x, y be any two vectors in H. Then

A) $||x + y||^2 + ||x - y||^2 = ||x||^2 + ||y||^2$

B) $|< x, y >| > ||x|| ||y||$

C) $2(||x + y||^2 + ||x - y||^2) = ||x||^2 + ||y||^2$

D) $x_n \rightarrow x$ and $y_n \rightarrow y$ implies $< x_n, y_n > \rightarrow < x, y >$

37. Let X be a normed linear space and x_o be a non zero vector in X. Then there exist a functional f_o in X' such that

A) $f_o(x_o) = x_o$ and $||f_o|| = 1$ C) $f_o(x_o) = 1$ and $||f_o|| = 1$

B) $f_o(x_o) = ||x_o||$ and $||f_o|| = 1$ D) $f_o(x_o) = 1$ and $||f_o|| \geq 1$

38. Let X and Y be a Banach space . If $f: X \rightarrow Y$ is a continuous linear transformation , then

A) T is closed B) T is open C) Range of T is finite dimensional D) None of these

39. Let X be a Banach algebra and $x \in X$, then the spectral radius is

A) $\lim_{n\to\infty} ||x^n||^{1/n}$ C) $\lim_{n\to\infty} \left\|x^{1/n}\right\|^{1/n}$

B) $\lim_{n\to\infty} \left\|x^{1/n}\right\|^{n}$ D) $\lim_{n\to\infty} ||x^n||^n$

40. The function $f(x) = x^2 - 2$ defined on the set of real numbers is

(A) injective but not surjective
(B) neither injective nor surjective
(C) surjective but not injective
(D) both injective and surjective

41. A man is watching from the top of a tower, a boat speeding away from the tower. The angle of depression from the top of the tower to the boat is 60^0 when the boat is $80m$ from the tower. After 10 seconds, the angle becomes 30^0. What is the speed of the boat?(Assume that the boat is running in still water)

(A) 20m/sec
(B) 10m/sec
(C) 18m/sec
(D) 16m/sec

42. The equation to the straight line which passes through the point $(-5, 4)$ and is such that the portion of it between the axes is divided by this point in the ratio $1 : 2$ is

(A) $5x + 8y = 7$
(B) $5x - 8y = -57$
(C) $5y + 8x = -20$
(D) $5y - 8x = 60$

43. The equation of the hyperbola whose vertices are at $(\pm 6, 0)$ and one of the directrices is $x = 4$ is

(A) $\frac{x^2}{45} - \frac{y^2}{36} = 1$
(B) $\frac{x^2}{36} - \frac{y^2}{45} = 1$
(C) $\frac{x^2}{25} - \frac{y^2}{36} = 1$
(D) none of these

44. $\lim_{x\to 1}(2 - x)^{\tan\frac{\pi x}{2}}$ is equal to

(A) $e^{1/\pi}$
(B) $e^{2/\pi}$
(C) $e^{3/\pi}$
(D) $\frac{2}{\pi}$

45. Equation to the normal to the curve $x^2 + y^2 = 5$ at the point $(2, 1)$ is

(A) $x - 2y = 0$
(B) $x + 2y = 0$
(C) $x - 2y = 3$
(D) $x + 2y = 3$

46. Area enclosed by the curve $27x^2 + 12y^2 - 324 = 0$ between the lines $x = 0$ and $x = 2\sqrt{3}$ is

(A) 7π
(B) 9π
(C) 2π
(D) $\frac{\pi}{2}$

47. A card is drawn from a well shuffled pack of 52 cards. The probability that the card drawn is a queen of clubs or a king of hearts is

(A) $\frac{1}{26}$ (B) $\frac{1}{52}$ (C) $\frac{1}{13}$ (D) $\frac{1}{2}$

48. Which among the following is a *false* statement ?

(A) Any bounded sequence of real numbers contains a convergent sub sequence.

(B) A sequence of real numbers is convergent if and only if it is a Cauchy sequence.

(C) If a is an accumulation point of a sequence $\{x_n\}_{n=1}^{\infty}$, then there is a sub sequence that converges to a.

(D) Any sequence $\{x_n\}_{n=1}^{\infty}$ is convergent if and only if it is bounded.

49. If a function f is monotonic on $[a, b]$, then the set of discontinuities of f is

(A) empty (B) finite (C) countable (D) $[a, b]$

50. Let A be the set of all rational numbers in the interval $[0, 1]$, and α be the Lebesgue measure of A, then α is equal to

(A) zero (B) one (C) infinity (D) none of these

51. The harmonic conjugate of the function $e^x \cos y + e^y \cos x + xy$ is

(A) $e^x \sin y - e^y \sin x + \frac{1}{2}(x^2 + y^2)$

(B) $e^x \sin y + e^y \sin x + \frac{1}{2}(x^2 + y^2)$

(C) $e^x \sin y + e^y \sin x - \frac{1}{2}(x^2 + y^2)$

(D) none of these

52. The Mobius transformation $T(z)$ that maps $z_1 = 1,\ z_2 = 0,\ z_3 = -1$ onto the points $w_1 = i,\ w_2 = \infty,\ w_3 = 1$ is

(A) $T(z) = \dfrac{(i-1)z + (i+1)}{2z}$

(B) $T(z) = \dfrac{(i+1)z + (i-1)}{2z}$

(C) $T(z) = \dfrac{(i-1)z - (i+1)}{2z}$

(D) none of these

53. The value of the integral $\displaystyle\int \frac{1}{z^2+4} dz$ around the circle $|z - i| = 2$ oriented in counter clockwise direction is

(A) zero (B) π (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

54. Let G be a group, $a \in G$ and $H = \{a^n | n \in \mathcal{Z}\}$ where $\mathcal{Z}$ is the set of integers. Then which of the following is *not* true?

(A) H is a subgroup of G

(B) G and H have the same identity

(C) H is the smallest subgroup of G containing the element a

(D) None of these

55. The number of abelian groups (up to isomorphism) of order 24 is

(A) 2 (B) 3 (C) 8 (D) None of these

56. Number of left cosets of the subgroup $<18>$ of $\mathcal{Z}_{36}$ is

(A) 18 (B) 36 (C) 4 (D) none of these

57. If U denotes the set of units in the ring of rational numbers $\mathcal{Q}$, then

(A) $U = \{1\}$

(B) $U = \{1, 2\}$

(C) U is empty

(D) U consists of all non-zero elements of $\mathcal{Q}$

58. The characteristic of the ring $\mathcal{C}$ of complex numbers is

(A) zero (B) one (C) infinity (D) none of these

59. The degree over $\mathcal{Q}$ of the splitting field over $\mathcal{Q}$ of the polynomial $x^2 + 3$ in $\mathcal{Q}[x]$ is

(A) Zero (B) 1 (C) 2 (D) none of these

60. If $A = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, then $(AB)^{-1}$ is equal to

(A) $\frac{1}{11}\begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$

(B) $\frac{1}{11}\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$

(C) $\frac{1}{11}\begin{pmatrix} 5 & 1 \\ 14 & 5 \end{pmatrix}$

(D) $\frac{1}{11}\begin{pmatrix} 14 & 5 \\ 5 & 1 \end{pmatrix}$

61. The value of the determinant $\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix}$ is equal to

(A) $(a^6 + b^6)$

(B) $(a^6 - b^6)$

(C) $(a^3 + b^3)^2$

(D) $(a^3 - b^3)^2$

62. If the vector $(3k+2, 3, 10)$ belongs to the linear span of the set $S = \{(-1, 0, 1), (2, 1, 4)\}$, then the value of k is

(A) 2 (B) -2 (C) 1 (D) -1

63. If the dimensions of the subspaces $\mathcal{U}$ and $\mathcal{V}$ of the vector space $\mathcal{W}$ are respectively 3 and 4 and $\dim(\mathcal{U} \cap \mathcal{V}) = 1$, then $\dim(\mathcal{U} + \mathcal{V})$ is equal to

(A) 4 (B) 6 (C) 7 (D) none of these

64. Which of the following is a subspace of the two dimensional Euclidean plane?

(A) $2x + 3y = 0$
(B) $2x + 3y + 1 = 0$
(C) $2x - 3y + 1 = 0$
(D) $2x + 3y - 1 = 0$

65. If $T : \mathcal{R}^3 \to \mathcal{R}^2$ is defined by $T(x, y, z) = (x, y)$, then the dimension of the kernel of T is

(A) 0 (B) 1 (C) 2 (D) indeterminate

66. The characteristic polynomial of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$ is

(A) $\lambda^3 + 6\lambda^2 - 11\lambda + 6$
(B) $\lambda^3 + 6\lambda^2 - 11\lambda - 6$
(C) $\lambda^3 - 6\lambda^2 + 11\lambda - 6$
(D) none of these

67. A matrix A is diagonalizable if the roots of its characteristic polynomial are

(A) real and equal
(B) real and distinct
(C) imaginary
(D) none of these

68. If $\gcd(a, b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right)$ is equal to

(A) $\frac{ab}{d}$ (B) d (C) d^2 (D) 1

69. The remainder when 97! (factorial) is divided by 101 is

(A) 15
(B) 16
(C) 17
(D) none of these

70. The differential equation of the family of all concentric circles centred at the origin is

(A) $y + x\dfrac{dy}{dx} = c$

(B) $y - x\dfrac{dy}{dx} = c$

(C) $x + y\dfrac{dy}{dx} = 0$

(D) none of these

71. The solution of the differential equation $(y^2 - y)dx + xdy = 0$ is

(A) $y(x + c) = x$

(B) $x(x + c) = y$

(C) $x(y + c) = x$

(D) none of these

72. Complete solution of the partial differential equation $p^2 + q^2 = m^2$ is

(A) $z = ax - y\sqrt{m^2 + a^2} + b$

(B) $z = ax + y\sqrt{m^2 + a^2} + b$

(C) $z = ax + y\sqrt{m^2 - a^2} + b$

(D) none of these

73. The partial differential equation
$\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} + 3\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y} = 0$ is

(A) parabolic

(B) elliptic

(C) hyperbolic

(D) none of these

74. In a metric space, every one point set is

(A) open

(B) closed

(C) both open and closed

(D) neither open nor closed

75. In the metric space $(\mathcal{R}^2, d_1)$, where $d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$, the sequence $\{\left(\dfrac{1}{n}, \dfrac{2n+1}{n+1}\right)\}$ converges to

(A) $(1, 0)$ (B) $(0, 1)$ (C) $(0, 2)$ (D) $(2, 0)$

76. In a topological space, which of the following is a *wrong* statement?

(A) Second countability is a hereditary property

(B) Metrizability is a hereditary property

(C) Regularity is a hereditary property

(D) None of these

77. Which of the following statements is *not* true?

(A) A subset of $\mathcal{R}$ is connected if and only if it is an interval

(B) Every closed and bounded interval is compact

(C) Closure of a connected subset is connected

(D) None of these

78. Let X be a normed linear space over the field K. E_1 and E_2 are non-empty disjoint convex subsets of X with E_1 open. Then there exist $f \in X'$ and $\alpha \in \mathcal{R}$, for all $x_1 \in E_1$ and $x_2 \in E_2$ such that

(A) $\text{Re}f(x_1) \leq \alpha \leq \text{Re}f(x_2)$

(B) $\text{Re}f(x_1) \leq \alpha < \text{Re}f(x_2)$

(C) $\text{Re}f(x_1) < \alpha < \text{Re}f(x_2)$

(D) $\text{Re}f(x_1) < \alpha \leq \text{Re}f(x_2)$

79. Let X and Y be Banach spaces and $B(X,Y)$ denotes the set of bounded linear maps from X to Y. Then which of the following statements is *not* true?

(A) Every closed linear map $A : X \to Y$ is continuous

(B) If $A \in B(X,Y)$ is surjective, then A is an open map

(C) If $A \in B(X,Y)$ is bijective, then $A^{-1} \in B(Y,X)$

(D) None of these

80. Let $\{u_1, u_2, \cdots, u_m\}$ be an orthonormal set in an inner product space X. Then for $x \in X$, $\Sigma_{n=1}^{m} |\langle x, u_n \rangle|^2 = \|x\|^2$ if and only if

(A) $x \in \{u_1, u_2, \cdots, u_m\}$

(B) $x \notin \{u_1, u_2, \cdots, u_m\}$

(C) $x \in \text{span}\{u_1, u_2, \cdots, u_m\}$

(D) $x \notin \text{span}\{u_1, u_2, \cdots, u_m\}$